



Hardy-Weinberg Problem Set

1. The frequency of two alleles in a gene pool is 0.19 (A) and 0.81(a). Assume that the population is in Hardy-Weinberg equilibrium.

(a) **Calculate** the percentage of heterozygous individuals in the population.

According to the Hardy-Weinberg Equilibrium equation, heterozygotes are represented by the $2pq$ term. Therefore, the number of heterozygous individuals (Aa) is equal to $2pq$ which equals $2 \times 0.19 \times 0.81 = 0.31$ or 31%

(b) **Calculate** the percentage of homozygous recessives in the population.

The homozygous recessive individuals (aa) are represented by the q^2 term in the H-W equilibrium equation which equals $0.81 \times 0.81 = 0.66$ or 66%

2. In a population that is in Hardy-Weinberg equilibrium, the frequency of the recessive homozygote genotype of a certain trait is 0.09. **Calculate** the percentage of individuals homozygous for the dominant allele.

We know that the frequency of the recessive homozygote genotype is q^2 and equal to 0.09.

Therefore $q = 0.30$ AND we also know that $p + q = 1$

Thus, $p = 1 - q$

$$p = 1 - 0.30 = 0.70$$

Therefore the homozygote *dominants* are represented by $p^2 = (0.70)^2 = 0.49$ or 49%

3. In a population that is in Hardy-Weinberg equilibrium, 38 % of the individuals are recessive homozygotes for a certain trait. In a population of 14,500, **calculate** the percentage of homozygous dominant individuals and heterozygous individuals.

Always start with the homozygous recessive percentage if given, which is equal to q^2 .

$$q^2 = 0.38 \text{ SO } q = 0.616$$

Solving for p is now straightforward: $p + q = 1$, thus $p = 0.384$

The homozygous dominant individuals are represented by $p^2 = 0.147$

So . . . 14,500 total population \times 0.0147 = 2,132 homozygous dominant individuals

The heterozygotes are represented by the $2pq$ term and $2 \times 0.616 \times 0.384 = 0.473$

So . . . 14,500 total population \times 0.473 = 6,859 heterozygous individuals

4. In humans, the *Rh* factor genetic information is inherited from our parents, but it is inherited independently of the ABO blood type alleles. In humans, *Rh*⁺ individuals have the *Rh* antigen on their red blood cells, while *Rh*⁻ individuals do not. There are two different alleles for the *Rh* factor known as *Rh*⁺ and *rh*. Assume that a dominant gene *Rh* produces the *Rh*⁺ phenotype, and that the recessive *rh* allele produces the *Rh*⁻ phenotype. In a population that is in Hardy-Weinberg equilibrium, 160 out of 200 individuals are *Rh*⁺. **Calculate** the frequency of both alleles.

If 160 of 200 individuals are *Rh*⁺, then it stands to reason that 40 are *Rh*⁻. Thus, the frequency of q^2 , the *Rh*⁻ genotype, is $40/200 = 0.20$ ($q^2 = 0.20$)
So ... $q = 0.45$ AND $p + q = 1$ so, $p = 0.55$
So ... Rh^+ allele frequency = 0.55 and rh allele frequency = 0.45

5. In corn, kernel color is governed by a dominant allele for white color (*W*) and by a recessive allele (*w*). A random sample of 100 kernels from a population that is in H-W equilibrium reveals that 9 kernels are yellow (*ww*) and 91 kernels are white.

(a) **Calculate** the frequencies of the yellow and white alleles in this population.
Always start with the homozygous recessive percentage if given, which is equal to q^2 .
The frequency of the yellow kernel recessive genotype *ww* is $9/100 = .09$ ($.09 = q^2$)
So ... $q = 0.30$ = the frequency of *w*
And $p + q = 1$ thus, $p = 0.70$ = the frequency of *W*

(b) **Calculate** the percentage of this population that is heterozygous.
The heterozygotes = $2pq = 2(0.70)(0.30) = 0.42$ or 42%

6. A rare disease which is due to a recessive allele (*a*) that is lethal when homozygous, occurs within a specific population at a frequency of one in a million. **Calculate** the number of individuals in a town having a population of 14,000 can be expected to carry this allele? (aka heterozygous!)

Always start with the homozygous recessive percentage if given, which is equal to q^2 .
The frequency of the recessive genotype, q^2 is $1/1,000,000 = 0.000001$

So ... $q = 0.001$ = the frequency of allele *a*

AND $p + q = 1$ thus, $p = 0.999$

Therefore, the frequency of allele *A* = $p = 0.999$ and the frequency of allele *a* = $q = 0.001$

Carriers are heterozygous and are equal to $2pq$. So, $2pq = 2(0.999)(0.001) = .001998$

And $.001998 \times 14000$ total population = 27.97

(This should be reported as 28 individuals carrying the allele upon considering the rounding done throughout the mathematics.)

Questions 7 & 8

Two Siamese and three Persian cats survive a shipwreck and are carried on driftwood to a previously uninhabited tropical island. All five cats have normal ears, but one carries the recessive allele f or folded ears (his genotype is Ff).

7. **Calculate** the frequencies of alleles F and f in the cat population of this island.

We are told that the genotype for the carrier is Ff . So, the other four cats are not carriers, thus their genotype for this trait is FF . Thus there are 10 total alleles within this population (5 cats x 2 alleles), 9 being F alleles and 1 being f .

So, the frequencies are 9/10 or 0.90 for F and 1/10 or 0.10 for the f allele

8. If you assume Hardy-Weinberg equilibrium for these alleles (admittedly very improbable), **calculate** the number of cats you would expect to have folded ears when the island population reaches 20,000?

The genotype of the cats with folded ears is ff which is equivalent to q^2 .

Therefore, $q^2 = (0.10)^2 = 0.010$ and the number of cats with folded ears is then equal to 20,000 total population $\times 0.010 =$ 200 cats with folded ears.

9. In a certain African population, 4 % of the population is born with sickle cell anemia (aa). **Calculate** the percentage of individuals who enjoy the selective advantage of the sickle-cell gene (increased resistance to malaria)?

The individuals that are heterozygous enjoy the selective advantage of increased resistance to malaria and are carriers of the sickle cell trait. They have a genotype of Aa and are represented by the $2pq$ term in the H-W equilibrium equations.

We are told that 4% of the population is affected with sickle cell anemia, thus $q^2 = 4\% = 0.04$ when expressed as a decimal. So, $q = 0.20$ AND $p + q = 1$, thus $p = 0.80$

Thus, the A allele has a frequency of 0.80 and the a allele has a frequency of 0.20. Therefore, the percentage of the population that is heterozygous (Aa) and are carriers is $= 2pq = 2(0.20)(0.80) = 0.32$ or 32%.

READY FOR THE FUN CHI- SQUARED PROBLEM? Deep breath and go!

10. In Caucasian humans, hair straightness or curliness is thought to be governed by a single pair of alleles showing partial dominance. Individuals with straight hair are homozygous for the *I_s* allele, while those with curly hair are homozygous for the *I_c* allele. Individuals with wavy hair are heterozygous (*I_s/I_c*). In a population of 1,000 individuals, 245 were found to have straight hair, 393 had curly hair, and 362 had wavy hair.

(a) **Calculate** the allelic frequencies of the *I_s* and *I_c* alleles.

Phenotype (genotype)	# <i>I_s</i> alleles	# <i>I_c</i> alleles
Straight (<i>I_sI_s</i>)	490	0
Wavy (<i>I_sI_c</i>)	362	362
Curly (<i>I_cI_c</i>)	0	786
Total	490+362 = 852 852/2000 (total alleles in pop) = .43 p=.43	362+786 = 1148 1148/ 2000 total alleles in pop= .57 q=.57

(b) **Explain** whether or not this population is in Hardy-Weinberg equilibrium? Justify your answer. Your explanation should include a chi-square goodness of fit test.

$$p^2 = .1849$$

$$2pq = .4902$$

$$q^2 = .3249$$

Individuals	Straight	Wavy	Curly
Expected	$p^2 \times 1000$ individuals = 185	$2pq \times 1000$ individuals = 490	$q^2 \times 1000$ individuals = 325
Observed (stated in problem)	245	362	393
(o-e)	60	-128	68

$$\chi^2 = \sum \frac{(o-e)^2}{e} = \left(\frac{60^2}{185} \right) + \left(\frac{-128^2}{490} \right) + \left(\frac{68^2}{325} \right) = 67.1 \text{ (with 2 degrees of freedom*)}$$

*The degrees of freedom equal $n - 1$, where n equals the number of genotypic classes, 3 in our case.

	Degrees of Freedom					
p	1	2	3	4	5	6
0.05	3.84	5.99	7.82	9.49	11.07	12.59
0.01	6.64	9.32	11.34	13.28	15.09	16.81

The null hypothesis is that the population *is* in Hardy-Weinberg Equilibrium. In order to reject that hypothesis, you must not only calculate Chi-square, but interpret the chi-square value using the table above considering two degrees of freedom and focusing on the probability (p) = 0.05 (95%) row.

Our calculated number of 67.1 is far greater than the critical value of 5.99 from the table, thus we **REJECT the null hypothesis** and conclude that our population **is not** in Hardy-Weinberg equilibrium. Thus, evolution is indeed taking place.